A Heuristic Solution Method to a Stochastic Vehicle Routing Problem

Lars M. Hvattum
University of Bergen, Bergen, Norway.
larsmh@ii.uib.no

Arne Løkketangen
Molde University College, 6411 Molde, Norway.
Arne.Lokketangen@himolde.no

Gilbert Laporte
Canada Research Chair in Distribution Management, HEC Montreal, Canada
gilbert@crt.umontreal.ca

Abstract

In order to solve real-world vehicle routing problems, the standard Vehicle Routing Problem (VRP) model usually needs to be extended. Here we consider the case when customers can call in orders during the daily operations, i.e. both customer locations and demand may be unknown in advance. Our heuristic approach attempts to minimize the expected number of vehicles and their travel distance in the final execution, under the condition that the unknown parameters have a known (approximated) distribution.

1 Introduction

The problem discussed in this paper is based on a real-world case, where a freight company is serving pick-up customers, i.e. customers who need goods to be picked up at their location and brought back to a central depot. The goods may then be sent to another location from the depot. The company is the largest freight company in Norway, and has provided us with real-world data. Although the company also handles delivery customers, we focus only on the pick-up part in this work.

Unfortunately, only a portion of the pick-up customers is known at the beginning of the day. During operation additional customers can call in their orders, and the freight company will modify its routes in order to serve the new customers. In current practice this often implies sending out additional vehicles.

A customer has a location and a time-window where service is accepted, but the time-windows are rather wide, often spanning the entire working day. In addition, each order has a weight, and the vehicle that services the customer must have sufficient capacity for supporting the pick-up load. For the deterministic customers we assume that the time-window and the weight of the order are known, whereas the number of stochastic customers, their call-in time, as well as the location, time-window, and capacity-demand of each individual stochastic customer, are only known probabilistically. The location, time-window and capacity-demand will, however, be revealed at call-in time.

In section 2 we formulate our problem formally. The heuristic solution method is described in section 3. Computational tests are discussed in section 4, followed by conclusions and suggestions for future work in section 5.

2 Problem description

In the Vehicle Routing Problem (VRP) a set of customers require some kind of service, which is offered by a fleet of vehicles. The goal is to find routes for the vehicles, each starting from a given depot to which they must return, such that every customer is visited exactly once. Usually there is also an objective that needs to be optimised, e.g. minimizing the travel cost or the number of vehicles needed.

The standard VRP is an NP-hard problem, and much work has been done in order to find efficient algorithms and heuristics to solve it. In real-world situations, however, the VRP usually fails to capture all essential aspects of the problem at hand. Hence, a large number of extensions have been studied, e.g. adding capacities, time windows, different kinds of customers, split deliveries, and more, which all try to make richer and more useful formulations.

Another way of extending the VPR is to focus on problems with uncertainty, where some of the parameters to the model are initially unknown or only known probabilistically. These approaches can roughly be divided in two classes: Stochastic Vehicle Routing Problems (SVRPs) and Dynamic Vehicle Routing Problems (DVRPs). Both classes have recently received an increasing amount of attention, and reviews can be found in Gendreau, Laporte and Séguin (1996a), Gendreau and Potvin (1998), and Ichoua et al. (2001).

The difference between static and dynamic VRPs is described in Psaraftis (1995), where the terms are defined as follows; a vehicle routing problem is *static* if the inputs to the problem do not change, neither during execution of the algorithm that solves it nor the execution of the solution. On the other hand, a problem is considered *dynamic* when inputs to the problem are made known or updated to the decision maker concurrently with the determination of the solution (i.e. the set of routes). In dynamic solution approaches no plan is generated a priori, and new events are handled as they are revealed over time.

A stochastic vehicle routing problem (SVRP) arises when some of the elements of the problem are stochastic. This could be relevant for many of the components that may be included in a standard VRP, for instance travel times, demands, existence of customers et cetera. SVRPs are usually formulated as two-stage stochastic programming problems. Then probabilistic information is used to construct an a priori plan, and recourse actions are defined to handle the situations that occur when the random variables are realized.

We would like to treat our problem as a mix between a DVRP and a SVRP. In our multi-stage formulation, we divide the time horizon into intervals. The goal is then to make a plan for how to serve customers within the next time interval, using known, deterministic information, as well as knowledge about distributions for the stochastic elements. This plan should be feasible over the whole (remaining) time horizon. Assuming that the routes followed in subsequent time-intervals are optimal, the plan should minimize the expected cost of serving all customers, stochastic as well as deterministic, before the end of the day. The cost of a plan must be calculated after all stochastic variables are realized, and is dependent on the number of vehicles used as well as the total travel distance.

Mathematical formulation

The deterministic, capacitated VRP with time-windows and a homogeneous fleet of vehicles can be formulated as a mixed integer program (MIP). Let the set of customers be denoted by $C = \{1,2,...,n\}$ and let 0 be the depot, so that the set of all locations is $N = C \cup \{0\}$. Travel times, c_{ij}, are given between every pair of locations $i, j \in N$, possibly including service time. Every customer i has a demand, d_i, and a time window for service, $[a_i, b_i]$. The time window for the depot is given by $[a_0, b_0]$ and, like the time windows for the customers, is considered hard – i.e. service must start after a_i and before b_i. (Since service time is included in c_{ij}, it must also be subtracted from the start and end of the corresponding time window.) A set of vehicles, $V = \{1,2,...,k\}$, each with a capacity q, start at the depot at a_0 and must be back at b_0, after servicing the customers.

The decision variables are x_{ijk}, where $x_{ijk} = 1$ if vehicle k travels from i to j, and $x_{ijk} = 0$ otherwise. In addition, s_{ik} denotes the time at which vehicle k has completed service at customer i. Note that the variables s_{ik} make designated sub-tour elimination constraints redundant. Then, based on the formulations used in Ho and Haugland (2002) and Kallehauge, Larsen and Madsen (2001), we can write the MIP as follows:

$$\min z(x,s) = \sum_{k\in V}\sum_{i\in N}\sum_{j\in N} c_{ij}x_{ijk} + K\sum_{k\in V}(1 - x_{00k}) \tag{1}$$

s.t.

$$\sum_{k\in V}\sum_{i\in N} x_{ijk} = 1 \qquad \forall j \in C \tag{2}$$

$$\sum_{j\in N} x_{0jk} = 1 \qquad \forall k \in V \tag{3}$$

$$\sum_{i\in N} x_{ihk} - \sum_{j\in N} x_{hjk} = 0 \qquad \forall h \in C, \forall k \in V \tag{4}$$

$$\sum_{i\in N} x_{i0k} = 1 \qquad \forall k \in V \tag{5}$$

$$\sum_{i\in C} d_i \sum_{j\in N} x_{ijk} \le q \qquad \forall k \in V \tag{6}$$

$$s_{ik} + c_{ij} - M_{ij}(1 - x_{ijk}) \le s_{jk} \quad \forall j \in C, \forall i \in N, \forall k \in V \quad (7)$$

$$a_i \le s_{ik} \le b_i \quad \forall i \in N, \forall k \in V \quad (8)$$

$$s_{ik} + c_{i0} - M_{i0}(1 - x_{i0k}) \le b_0 \quad \forall i \in C, \forall k \in V \quad (9)$$

$$s_{0k} = a_0 \quad \forall k \in V \quad (10)$$

$$x_{iik} = 0 \quad \forall i \in C, \forall k \in V \quad (11)$$

$$x_{ijk} \in \{0,1\} \quad \forall i, j \in N, \forall k \in V \quad (12)$$

Here, the objective (1) is to minimize the number of vehicles used plus the total travel time. The constant K represents the cost of starting an extra vehicle. Constraints (3), (4) and (5) states that each vehicle should leave from the depot once, leave each customer that it arrives, and arrive back at the depot once. Inequality (6) assures that the capacity of the vehicle is not violated. While constraint (7) ensures that vehicle k spend the correct amount of time driving between i and j before servicing customer j, inequalities (8) corresponds to the time windows for each customer. In addition (9) force every vehicle to arrive at the depot before the end of the depot's time window. The constant M_{ij} is defined to be sufficiently large, e.g. $M_{ij} = b_i + c_{ij} - a_j$.

In the two-stage stochastic counterpart, assume that there exists some single known point in time, $t \in [a_0, b_0]$, where all previously unknown information is revealed. That is, all stochastic customers, their location, demand and time windows become known. We can model this as a two-stage stochastic program, where we introduce new variables to handle recourse actions.

Let $C^t = \{n+1, n+2, ..., n+n^t\}$ be the set of revealed customers. Let d_i and $[a_i, b_i]$ be the demand and time-window for the customers, $i \in C^t$. Let the set of all locations be $N' = N \cup C'$, where $C' = C \cup C^t$. The travel time between i and j is c_{ij}, $i, j \in N'$. Let $\xi = (C^t, D^t, A^t, B^t, c^t)$ be a particular realization of the random variables $\tilde{\xi} = (\tilde{C}^t, \tilde{D}^t, \tilde{A}^t, \tilde{B}^t, \tilde{c}^t)$.

The two-stage stochastic programming problem can then be formulated as

$$\min E_{\tilde{\xi}}\left[z(x,s) + Q(x,s,\tilde{\xi})\right] \quad (13)$$

where $$Q(x,s,\tilde{\xi}) = \sum_{i \in N'} \sum_{j \in N'} \sum_{k \in V} c_{ij}\left(x_{ijk}^{t+} - x_{ijk}^{t-}\right) + K \sum_{k \in V} (x_{00k}^{t-} - x_{00k}^{t+}) \quad (14)$$

s.t. (2)-(12)

$$\sum_{k\in V}\sum_{i\in N'}\left(x_{ijk}+x_{ijk}^{t+}-x_{ijk}^{t-}\right)=1 \qquad \forall j\in C' \qquad (15)$$

$$\sum_{j\in N'}\left(x_{0jk}+x_{0jk}^{t+}-x_{0jk}^{t-}\right)=1 \qquad \forall k\in V \qquad (16)$$

$$\sum_{i\in N'}\left(x_{ihk}+x_{ihk}^{t+}-x_{ihk}^{t-}\right)-\sum_{j\in N'}\left(x_{hjk}+x_{hjk}^{t+}-x_{hjk}^{t-}\right)=0 \qquad \forall h\in C',\forall k\in V \qquad (17)$$

$$\sum_{i\in N'}\left(x_{i0k}+x_{i0k}^{t+}-x_{i0k}^{t-}\right)=1 \qquad \forall k\in V \qquad (18)$$

$$\sum_{i\in C}d_i\sum_{j\in N}(x_{ijk}+x_{ijk}^{t+}-x_{ijk}^{t-})\le q \qquad \forall k\in V \qquad (19)$$

$$x_{ijk}^{t-}\le x_{ijk} \qquad \forall i,j\in N,\forall k\in V \qquad (20)$$

$$x_{ijk}^{t-}=0 \qquad \forall i,j\in C^t,\forall k\in V \qquad (21)$$

$$tx_{ijk}^{t-}\le s_{ik} \qquad \forall i,j\in N,\forall k\in V \qquad (22)$$

$$s_{ik}^t+c_{ij}-M_{ij}(1-x_{ijk}-x_{ijk}^{t+}+x_{ijk}^{t-})\le s_{jk}^t \qquad \forall j\in C',\forall i\in N',\forall k\in V \qquad (23)$$

$$a_i\le s_{ik}^t\le b_i \qquad \forall i\in N',\forall k\in V \qquad (24)$$

$$s_{ik}^t+c_{i0}-M_{i0}(1-x_{i0k}-x_{i0k}^{t+}+x_{i0k}^{t-})\le b_0 \qquad \forall i\in C',\forall k\in V \qquad (25)$$

$$s_{0k}^t=a_0+\left(t-a_0\right)x_{00k} \qquad \forall k\in V \qquad (26)$$

$$x_{iik}^{t+}=0 \qquad \forall i\in C',\forall k\in V \qquad (27)$$

$$x_{ijk}^{t+}\in\{0,1\} \qquad \forall i,j\in N',\forall k\in V \qquad (28)$$

$$x_{ijk}^{t-}\in\{0,1\} \qquad \forall i,j\in N',\forall k\in V \qquad (29)$$

Here, x_{ijk}^{t+}, x_{ijk}^{t-} and s_{ik}^t depend on the random vector $\tilde{\xi}$, and thus are random variables themselves. $E_{\tilde{\xi}}$ stands for the expected value with respect to the distribution of $\tilde{\xi}$. $Q(x,s,\tilde{\xi})$ is the recourse cost, and sums up the cost of starting new vehicles and the change in travel time. $x_{ijk}^{t+}=1$ if vehicle k travels from i to j in second-stage recourse but not in first-stage solution, and $x_{ijk}^{t-}=1$ if vehicle k travelled from i to j in first-stage solution but not after recourse. s_{ik}^t is the time at which vehicle k has completed service at customer i according to the second-stage recourse actions.

Constraints (15)-(18) are similar to (2)-(5), ensuring that all customers are served, that any vehicle must leave the depot once, leave all customers that are visited, and arrive at the depot once. (19) ensures that vehicle capacities are not violated in second-stage solution. In (20)-(22) we state that a trip between customer i and j in the first-stage solution cannot be skipped in second-stage recourse if service at departure-location is completed before time t. A vehicle k that does not leave the depot in the first-stage solution ($x_{00k} = 1$) cannot leave the depot before t in a second-stage decision, as stated in (26). Constraints (23)-(25) are used to track the time at which service is completed in the final, second-stage solution.

As stated before, the call-in time for each stochastic customer is unknown. Thus, our two-stage model does not sufficiently capture the dynamic element of the real-world case upon which it is based. A possible remedy would be to treat the number of stages in the problem as a stochastic variable, with one stage per call-in. In this work, however, we opt for a simpler approach, in which we divide the time-horizon into a specified number of intervals (or stages), $I_1,...,I_n$. The objective is then to find a plan for the rest of the day, using the currently known customers. This is repeated dynamically for every time-interval, each time modifying the problem with information received during the previous interval. We omit the details of this multi-stage formulation, and direct the reader to Kall and Wallace (1994) for more on stochastic formulations.

3 The Heuristic

Exact solution methods for stochastic VRPs currently fail to consistently solve problems with more than just a few customers (see Gendreau, Laporte and Séguin, 1995 and 1996b). In our case, even evaluation of the recourse cost function (14) can be extremely difficult, depending on the distribution of the random variables. Hence, the need for practical heuristic solution methods is evident.

Our approach is based on solving sample-scenarios, where the idea is to use common features from sample-solutions to build a hopefully good plan. The term sample-scenario is here used to describe one of many potential future situations, i.e. actual realizations of customer call-ins. A sample-solution is then a plan that would be possible to execute given that the scenario would actually happen. The approach requires that we are able to generate possible future scenarios based on current deterministic information as well as approximations of the distributions of the stochastic variables involved. The idea of sampling in stochastic problems can be traced to Jagannathan (1985).

Assume we have divided the time horizon into n intervals, $I_1,...,I_n$. At the point in time corresponding to the beginning of interval I_u, a plan is constructed in the following manner:

Let C be the set of known customers at the start of I_u, and let S be a subset of C, initially empty.

1. Create p sample scenarios with the same known customers as in C, including stochastic customers drawn from the given distributions.
2. Repeat the following:
   2.1. Solve the sample scenarios, while forcing all customers in S to be served during I_u.
   2.2. Find the customer $c \in C - S$ that is most frequently visited in the time interval I_u in the p sample scenarios.
   2.3. If some stop criteria is met then go to 3, otherwise let $S = S \cup \{c\}$.
3. Repeat until S is empty:
   3.1. Solve the sample scenarios
   3.2. Count the number of times each customer $c \in S$ is served first by each vehicle in each scenario solution, disregarding customers that have already been placed.
   3.3. Choose the customer/vehicle-pair that has the highest count. Lock the customer to be served before all remaining customers by the given vehicle, and remove it from S.

Terminate with a plan for the given time interval, I_u.

After the end of I_u has passed the procedure is repeated, possibly including new customers that have called in their order. Customers served during the previous time intervals can now be ignored in the solution process, as long as the position and spare capacity of each vehicle is remembered.

In phase 1 we exploit knowledge about the probability distributions of the stochastic variables of the model, i.e. distributions of call-in times, demands, geographical locations, et cetera, to create possible future events. When including the known customers and their attributes, this constitutes our sample scenarios. They have the property that events that are likely to happen are present in a scenario with a high probability.

The second phase produces solutions of the sample scenarios, as if they were deterministic VRPs. Customers that are frequently served during I_u are then identified, and a decision is made to visit these customers during the next time interval in our final plan. Sample scenarios where these customers were not already served in I_u are then solved again, before the selection procedure continues. The selection is considered complete when there are no unselected customers that are frequently handled during I_u in solutions to the sample scenarios. We require that a customer must be handled early in at least half of the sample solutions in order to be selected, which in this case constitutes our stop criterion.

After phase two is completed we have found which customers to serve in the next time interval, but still have to decide in what sequence they should be visited. A tempting approach is to simply solve a deterministic VRP with the selected customers; however, we would rather exploit the information that is contained in the sample scenario solutions. Therefore we insert customers into the final plan depending on their positions in the sample plans; the customer most frequently visited first by some vehicle is locked to always be visited first by that vehicle. This corresponds to phase 3 in the procedure.

The heuristic has been implemented in C++, using a commercially available VRP-solver called SPIDER, produced by SINTEF, to generate heuristic solutions to the sample scenarios. In our heuristic we do not solve the sample scenarios very accurately, but use a simple insertion-neighbourhood to insert customers consecutively into the plan at the locally best location.

Note that this heuristic could be easily adapted to different types of stochastic VRPs, and that the underlying model could be extended in many directions.

4 Computational Results

In order to assess the quality of our heuristic, we have performed some tests using random generated test cases. The cases are based on real-world data, which was collected for other purposes (see Wahl et al., 2002). This data was used to generate approximated distributions for the stochastic variables in the model, which is then used to generate sample scenarios in the heuristic framework, as well as generating the test cases.

Table 1 – Computational results

		Deterministic		**Pure dynamic**		**SSBHH**		
Problem	**#Orders**	Total distance	No. of vehicles	Total distance	No. of vehicles	Total distance	No. of vehicles	Reduction of distance in %
1	142	147238	6	215030	6	205968	6	4.2%
2	126	111240	4	164724	4	122974	4	25.3%
3	147	144471	5	211902	5	158081	5	25.4%
4	118	122244	4	216632	4	141643	5	34.6%
5	126	134448	5	205136	5	158022	5	23.0%
6	118	113658	4	167514	4	133895	4	20.1%
7	128	124070	5	178059	5	146204	6	17.9%
8	129	149118	5	237306	5	187675	6	20.9%
9	117	97383	3	165345	3	105791	3	36.0%
10	127	138753	5	182529	5	168150	6	7.9%
11	146	125659	4	211174	4	139488	4	33.9%
12	151	121650	4	194143	4	136249	4	29.8%
13	138	104251	3	190020	3	118980	4	37.4%
14	126	122230	4	198243	4	137376	4	30.7%
15	139	128013	5	214064	5	153354	5	28.4%
16	122	117193	4	190670	4	128604	4	32.6%
17	125	143836	5	196545	5	146684	5	25.4%
18	143	155267	6	227041	6	185113	6	18.5%
19	144	138963	5	218846	5	155686	5	28.9%
20	139	143309	5	207547	5	167465	5	19.3%
Avg.	**132.55**	**129150**	**4.55**	**199624**	**4.55**	**149870**	**4.80**	**24.9%**

The results obtained by running the heuristic using 30 sample scenarios at each stage, with 16 stages for each of 20 test cases, is shown in table 1 under the heading 'SSBHH' ('Sample Scenario Based Hedging Heuristic'). The total running time per test case was about 6 minutes.

For comparison we have also presented, under the heading 'Deterministic', results achieved when considering all information as known a priori; that is, the cases are solved as deterministic VRPs using the SPIDER software. Each of these solutions was obtained by running an initial insertion heuristic followed by a variable neighbourhood local search, and the total running time per instance was approximately 35 minutes.

We have also tried to solve the problems using a myopic, dynamic approach, without using stochastic information, where at each stage we make a solution using currently known information only. This is presented under the heading 'Pure dynamic'. The computational effort used to solve the sub-problem at each stage was set such that the total running time of this approach would match the running time of the SSBHH.

It should be noted that the effect of the running time with respect to the execution of the resulting plan is ignored in both dynamic approaches, i.e. it is assumed that all information arriving up to the start of a time-interval can be used and that the plan generated at that point is immediately implemented. In practice one would have to reserve some time for the solver to run, by letting information that arrives during the planning to be handled in the next stage.

We observe that we can reduce the total travel distance quite substantially by applying our heuristic, as compared to the pure dynamic method. However, this does not come for free, but at the cost of sometimes having to use an extra vehicle. It should be noted that the number of vehicles may be a strategic rather than a day-to-day decision, and thus only the maximum number of vehicles used is important.

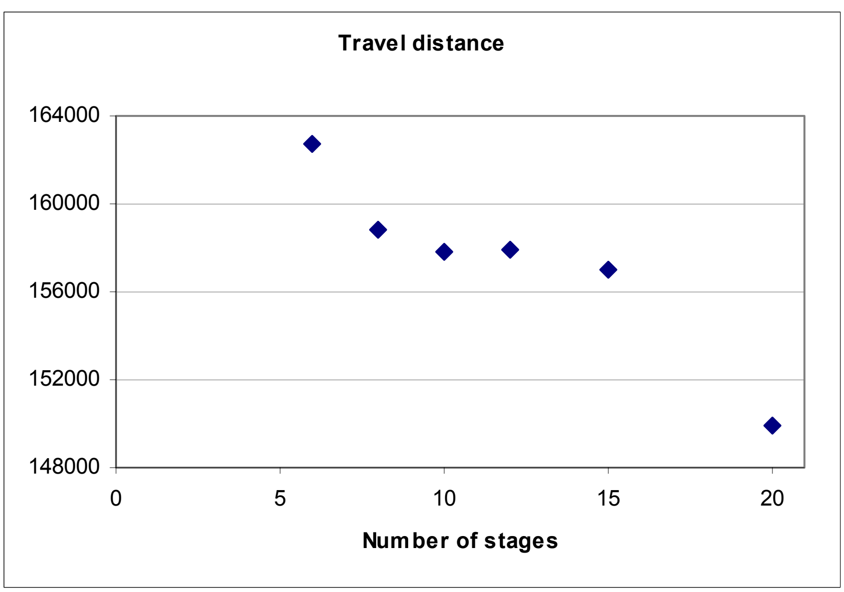

Figure 1 – Varying the number of stages of the model

We should also note that our heuristic seems to have the nice property of consistently producing better solutions when the number of sample scenarios at each stage is increased, when the number of stages is increased, or when more effort is spent on producing good solutions to the sample scenarios. This is illustrated in figures 1 and 2, where travel distance is plotted against the number of stages and the number of scenarios respectively. The number of vehicles used in the final execution is also reduced when increasing the

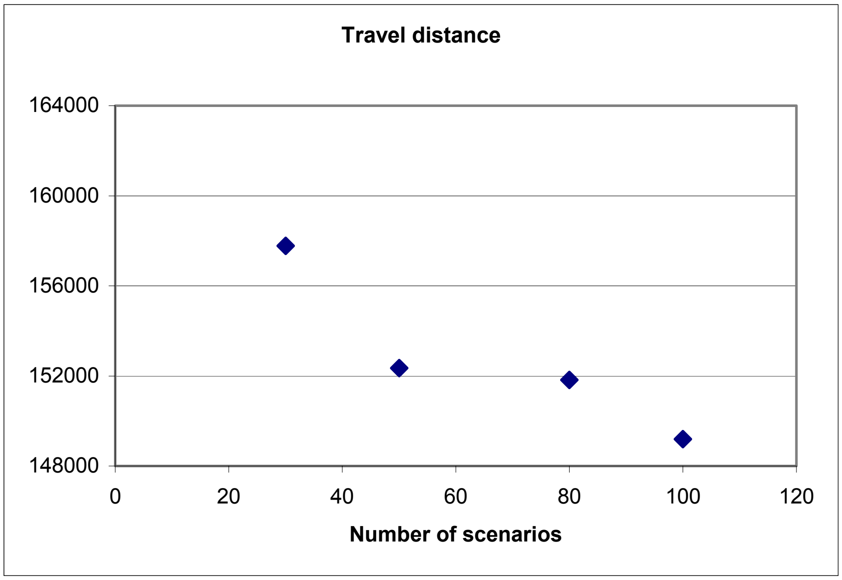

Figure 2 – Varying the number of sample scenarios at each stage

effort, although we suspect other parameters of the heuristic to have greater importance here, in a trade-off between vehicle utilization and travel distance.

Additional testing has shown that the same property is not present when increasing computational effort in the pure dynamic approach, where stochastic information is ignored. The interpretation here is that a better solution to the static sub-problems does not automatically yield better overall performance, as decisions that look good early may turn out to be inferior when new customers appear.

Additional testing has shown that increasing the computational effort does not as a rule yield better solutions in the pure dynamic approach. Our interpretation for this phenomenon is that when stochastic information is ignored in the solution of the subproblems, decisions that looked good at an early stage may turn out to be inferior when new customers appear. Thus, overoptimizing the subproblem represents a wasted effort.

5 Conclusions and future work

To solve realistic routing problems, it is often essential to extend the classical VRP formulation. Here we consider one particular instance of uncertainty, where customers may appear at unknown locations with random demand at any time during the given time-horizon.

We propose a rather general heuristic solution method, inspired partly by the idea of progressive hedging (see Haugen, Løkketangen and Woodruff, 2000), where sample scenarios are generated and their solutions inspected to find promising structures.

The computational findings so far indicate that this may lead to better solutions than pure dynamic approaches.

Several parameters in the heuristic could be tuned, which may lead to other insights. Parameters that need to be investigated further include when to stop including customers, how to insert them into the final plan and how many stages to use. For the latter we must also take computational effort into consideration, as there clearly is a limit on available resources when planning in a dynamic, real world, environment. However, there could be ample possibilities to use parallel computing during all phases of our heuristic, in case one wants to increase the number of stages.

We would also like to expand our stochastic model to support other stochastic variables, for instance stochastic travel times and stochastic demands for known customers, as well as add other extensions to the model, e.g. heterogeneous vehicles and dynamic travel times. This would allow for a wide range of dynamic stochastic VRPs to be solved using the same framework.

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